**Today’s Objective:**

Students will be able to:

a) Resolve a 2-D vector into components.

b) Add 2-D vectors using Cartesian vector notations.

**In-Class activities:**

- Check Homework
- Reading Quiz
- Application of Adding Forces
- Parallelogram Law
- Resolution of a Vector Using Cartesian Vector Notation (CVN)
- Addition Using CVN
- Attention Quiz
READING QUIZ

1. Which one of the following is a scalar quantity?
   A) Force  B) Position  C) Mass  D) Velocity

2. For vector addition, you have to use _______ law.
   A) Newton’s Second
   B) the arithmetic
   C) Pascal’s
   D) the parallelogram
APPLICATION OF VECTOR ADDITION

There are three concurrent forces acting on the hook due to the chains.

We need to decide if the hook will fail (bend or break).

To do this, we need to know the resultant or total force acting on the hook as a result of the three chains.
# SCALARS AND VECTORS

(Section 2.1)

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<td><strong>Characteristics:</strong></td>
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<td>It has a magnitude (positive or negative)</td>
<td>It has a magnitude and direction</td>
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In these PowerPoint presentations, a vector quantity is represented *like this* (in **bold**, *italics*, and *green*).
VECTOR OPERATIONS
(Section 2.2)

Scalar Multiplication and Division
Type of Forces

1. Coplanar concurrent
2. Coplanar non-concurrent
3. Coplanar parallel
4. Non-coplanar concurrent
5. Non-coplanar non-concurrent
6. Non-coplanar parallel

(a) Concurrent  (b) Nonconcurrent  (c) Parallel
ORTHOGONAL CONCURRENT FORCES: RESULTANTS AND COMPONENTS
Example 1:

The rectangular components of a force are +100 lb in the Y direction and -200 lb in the X direction. Both forces act through point O. Calculate the magnitude, inclination with the X axis, and the sense of the resultant force P.
Example 2:

Two forces, Fx and 4000 lb, intersect at O on a heavy object that is to be pulled along a line defined by a 40° angle with the Y axis. Determine the required force Fx and find the magnitude of the resultant of the two forces.
ADDITION OF A SYSTEM OF COPLANAR FORCES (Section 2.4)

- We ‘resolve’ vectors into components using the x and y-axis coordinate system.

- Each component of the vector is shown as a magnitude and a direction.

- The directions are based on the x and y axes. We use the “unit vectors” $i$ and $j$ to designate the x and y-axes.
NON-ORTHOGONAL CONCURRENT FORCES: RESULTANTS AND COMPONENTS
VECTOR ADDITION USING EITHER THE PARALLELOGRAM LAW OR TRIANGLE

Parallelogram Law:

Triangle method (always ‘tip to tail’):

How do you subtract a vector?

How can you add more than two concurrent vectors graphically?
Example 3

Determine the resultant \( R \) of the two-force coplanar system. Compute its magnitude, sense, and direction.
Example 4

Determine the resultant $R$ of the four-force coplanar force system. Compute its magnitude, sense, and direction.
"Resolution" of a vector is breaking up a vector into components.

It is kind of like using the parallelogram law in reverse.
For example,

\[ \mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j} \quad \text{or} \quad \mathbf{F'} = F'_x \mathbf{i} + (-F'_y) \mathbf{j} \]

The x and y axis are always perpendicular to each other. Together, they can be directed at any inclination.
ADDITION OF SEVERAL VECTORS

• Step 1 is to resolve each force into its components.

• Step 2 is to add all the x-components together, followed by adding all the y-components together. These two totals are the x and y-components of the resultant vector.

• Step 3 is to find the magnitude and angle of the resultant vector.
An example of the process:

Break the three vectors into components, then add them.
You can also represent a 2-D vector with a magnitude and angle.

\[ \theta = \tan^{-1} \left| \frac{F_{Ry}}{F_{Rx}} \right| \]

\[ F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} \]
Example 5

Determine the resultant \( R \) of the four-force coplanar force system. Compute its magnitude, sense, and direction. Use components method.
EXAMPLE 6

**Find:** The magnitude and angle of the resultant force.
EXAMPLE 6 (continued)
EXAMPLE 6 (continued)
CONCEPT QUIZ

1. Can you resolve a 2-D vector along two directions, which are not at 90° to each other?
   A) Yes, but not uniquely.
   B) No.
   C) Yes, uniquely.

2. Can you resolve a 2-D vector along three directions (say at 0, 60, and 120°)?
   A) Yes, but not uniquely.
   B) No.
   C) Yes, uniquely.
GROUP   PROBLEM  SOLVING 1

**Find:** The magnitude and angle of the resultant force.

![Diagram showing vectors](image-url)
GROUP PROBLEM SOLVING (continued)

\[ F_2 = 600 \text{ N} \]
\[ F_1 = 800 \text{ N} \]
\[ F_3 = 650 \text{ N} \]
ATTENTION QUIZ

1. Resolve \( \mathbf{F} \) along \( x \) and \( y \) axes and write it in vector form. \( \mathbf{F} = \{ \text{__________} \} \) N
   - A) \( 80 \cos (30^\circ) \mathbf{i} - 80 \sin (30^\circ) \mathbf{j} \)
   - B) \( 80 \sin (30^\circ) \mathbf{i} + 80 \cos (30^\circ) \mathbf{j} \)
   - C) \( 80 \sin (30^\circ) \mathbf{i} - 80 \cos (30^\circ) \mathbf{j} \)
   - D) \( 80 \cos (30^\circ) \mathbf{i} + 80 \sin (30^\circ) \mathbf{j} \)

2. Determine the magnitude of the resultant \((\mathbf{F}_1 + \mathbf{F}_2)\) force in N when \( \mathbf{F}_1 = \{ 10 \mathbf{i} + 20 \mathbf{j} \} \) N and \( \mathbf{F}_2 = \{ 20 \mathbf{i} + 20 \mathbf{j} \} \) N.
   - A) 30 N
   - B) 40 N
   - C) 50 N
   - D) 60 N
   - E) 70 N
Today’s Objectives:
Students will be able to:

a) Represent a 3-D vector in a Cartesian coordinate system.
b) Find the magnitude and coordinate angles of a 3-D vector
c) Add vectors (forces) in 3-D space

In-Class Activities:
• Reading Quiz
• Applications / Relevance
• A Unit Vector
• 3-D Vector Terms
• Adding Vectors
• Concept Quiz
• Examples
• Attention Quiz
1. Vector algebra, as we are going to use it, is based on a __________ coordinate system.
   A) Euclidean        B) Left-handed
   C) Greek           D) Right-handed  E) Egyptian

2. The symbols $\alpha$, $\beta$, and $\gamma$ designate the __________ of a 3-D Cartesian vector.
   A) Unit vectors    B) Coordinate direction angles
   C) Greek societies D) X, Y and Z components
Applications

Many structures and machines involve 3-dimensional space.

In this case, the power pole has guy wires helping to keep it upright in high winds. How would you represent the forces in the cables using Cartesian vector form?
In the case of this radio tower, if you know the forces in the three cables, how would you determine the resultant force acting at D, the top of the tower?
For a vector $\mathbf{A}$, with a magnitude of $A$, an unit vector is defined as

$$u_A = \frac{\mathbf{A}}{A}.$$ 

Characteristics of a unit vector:

a) Its magnitude is 1.

b) It is dimensionless (has no units).

c) It points in the same direction as the original vector ($\mathbf{A}$).

The unit vectors in the Cartesian axis system are $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$. They are unit vectors along the positive x, y, and z axes respectively.
CARTESIAN VECTOR REPRESENTATION

Consider a box with sides AX, AY, and AZ meters long.
Using trigonometry, “direction cosines” are found using

These angles are not independent. They must satisfy the following equation.

This result can be derived from the definition of a coordinate direction angles and the unit vector. Recall, the formula for finding the unit vector of any position vector:

or written another way
ADDITION OF CARTESIAN VECTORS

Once individual vectors are written in Cartesian form, it is easy to add or subtract them. The process is essentially the same as when 2-D vectors are added.

\[
F_R = \Sigma F = \Sigma F_x \mathbf{i} + \Sigma F_y \mathbf{j} + \Sigma F_z \mathbf{k}
\]

For example, if

\[
A = AX \mathbf{i} + AY \mathbf{j} + AZ \mathbf{k}
\]

and

\[
B = BX \mathbf{i} + BY \mathbf{j} + BZ \mathbf{k},
\]

then

\[
A + B = (AX + BX) \mathbf{i} + (AY + BY) \mathbf{j} + (AZ + BZ) \mathbf{k}
\]

or

\[
A - B = (AX - BX) \mathbf{i} + (AY - BY) \mathbf{j} + (AZ - BZ) \mathbf{k}.
\]
IMPORTANT NOTES

Sometimes 3-D vector information is given as:

a) Magnitude and the coordinate direction angles, or,
b) Magnitude and projection angles.

You should be able to use both these sets of information to change the representation of the vector into the Cartesian form, i.e.,

\[ \mathbf{F} = \{10 \mathbf{i} - 20 \mathbf{j} + 30 \mathbf{k}\} \text{ N.} \]
EXAMPLE 7

Given: Two forces \( F_1 \) and \( F_2 \) are applied to a hook.

Find: The resultant force in Cartesian vector form.

Plan:
EXAMPLE 7

(continued)
EXAMPLE 7
(continued)
EXAMPLE 7

(continued)
CONCEPT QUESTIONS

1. If you know only \( uA \), you can determine the ________ of \( A \) uniquely.
   
   A) magnitude  B) angles (\( \alpha \), \( \beta \), and \( \gamma \))  
   C) components (AX, AY, & AZ)  D) All of the above.

2. For a force vector, the following parameters are randomly generated. The magnitude is 0.9 N, \( \alpha = 30^\circ \), \( \beta = 70^\circ \), \( \gamma = 100^\circ \). What is wrong with this 3-D vector?
   
   A) Magnitude is too small.  
   B) Angles are too large.  
   C) All three angles are arbitrarily picked.  
   D) All three angles are between 0^\circ \) to 180^\circ \).
**GROUP PROBLEM SOLVING 2**

**Given:** The screw eye is subjected to two forces, $F_1$ and $F_2$.

**Find:** The magnitude and the coordinate direction angles of the resultant force.

**Plan:**

![Diagram showing forces and angles](image)
ATTENTION QUIZ

1. What is not true about an unit vector, e.g., \( uA \)?
   A) It is dimensionless.
   B) Its magnitude is one.
   C) It always points in the direction of positive X-axis.
   D) It always points in the direction of vector \( A \).

2. If \( F = \{10 \ i + 10 \ j + 10 \ k\} \) N and
   \( G = \{20 \ i + 20 \ j + 20 \ k\} \) N, then
   \[ F + G = \{ \ \ \} \ N \]
   A) \( 10 \ i + 10 \ j + 10 \ k \)
   B) \( 30 \ i + 20 \ j + 30 \ k \)
   C) \( -10 \ i - 10 \ j - 10 \ k \)
   D) \( 30 \ i + 30 \ j + 30 \ k \)
POSITION VECTORS & FORCE VECTORS

Today’s Objectives:
Students will be able to:

a) Represent a position vector in Cartesian coordinate form, from given geometry.
b) Represent a force vector directed along a line.

In-Class Activities:
- Check Homework
- Reading Quiz
- Applications/Relevance
- Write Position Vectors
- Write a Force Vector along a line
- Concept Quiz
- Group Problem
- Attention Quiz
READING QUIZ

1. The position vector $r_{PQ}$ is obtained by
   A) Coordinates of Q minus coordinates of the origin.
   B) Coordinates of P minus coordinates of Q.
   C) Coordinates of Q minus coordinates of P.
   D) Coordinates of the origin minus coordinates of P.

2. A force of magnitude F, directed along a unit vector $U$, is given by
   $F = ______$.
   A) $F \cdot U$
   B) $U / F$
   C) $F / U$
   D) $F + U$
   E) $F - U$
APPLICATIONS

This ship’s mooring line, connected to the bow, can be represented as a Cartesian vector.

What are the forces in the mooring line and how do we find their directions?

Why would we want to know these things?
This awning is held up by three chains. What are the forces in the chains and how do we find their directions? Why would we want to know these things?
A position vector is defined as a fixed vector that locates a point in space relative to another point.

Consider two points, A and B, in 3-D space. Let their coordinates be \((X_A, Y_A, Z_A)\) and \((X_B, Y_B, Z_B)\), respectively.
The position vector directed from A to B, \( \mathbf{r}_{AB} \), is defined as

\[
\mathbf{r}_{AB} = ((X_B - X_A) \mathbf{i} + (Y_B - Y_A) \mathbf{j} + (Z_B - Z_A) \mathbf{k}) \text{m}
\]

Please note that B is the ending point and A is the starting point. **ALWAYS** subtract the “tail” coordinates from the “tip” coordinates!
FORCE VECTOR DIRECTED ALONG A LINE (Section 2.8)

If a force is directed along a line, then we can represent the force vector in Cartesian coordinates by using a unit vector and the force’s magnitude. So we need to:

a) Find the position vector, \( \mathbf{r}_{AB} \), along two points on that line.
b) Find the unit vector describing the line’s direction, \( \mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{||\mathbf{r}_{AB}||} \).
c) Multiply the unit vector by the magnitude of the force, \( \mathbf{F} = F \mathbf{u}_{AB} \).
EXAMPLE 8

**Given:** The 420 N force along the cable AC.

**Find:** The force $\mathbf{F}_{AC}$ in the Cartesian vector form.
EXAMPLE 8 (continued)
CONCEPT QUIZ

1. **P** and **Q** are two points in a 3-D space. How are the position vectors \( \mathbf{r}_{\mathbf{PQ}} \) and \( \mathbf{r}_{\mathbf{QP}} \) related?

   A) \( \mathbf{r}_{\mathbf{PQ}} = \mathbf{r}_{\mathbf{QP}} \)  
   B) \( \mathbf{r}_{\mathbf{PQ}} = -\mathbf{r}_{\mathbf{QP}} \)  
   C) \( \mathbf{r}_{\mathbf{PQ}} = 1/\mathbf{r}_{\mathbf{QP}} \)  
   D) \( \mathbf{r}_{\mathbf{PQ}} = 2 \mathbf{r}_{\mathbf{QP}} \)

2. If **F** and \( \mathbf{r} \) are force and position vectors, respectively, in SI units, what are the units of the expression \( \mathbf{r} \times (\mathbf{F} / \mathbf{F}) \)?

   A) Newton  
   B) Dimensionless  
   C) Meter  
   D) Newton - Meter  
   E) The expression is algebraically illegal.
GROUP PROBLEM SOLVING 3

Given: Two forces are acting on a flag pole as shown in the figure. $F_B = 700 \text{ N}$ and $F_C = 560 \text{ N}$

Find: The magnitude and the coordinate direction angles of the resultant force.
ATTENTION QUIZ

1. Two points in 3–D space have coordinates of P (1, 2, 3) and Q (4, 5, 6) meters. The position vector $\mathbf{r}_{QP}$ is given by
   A) $\{3 \mathbf{i} + 3 \mathbf{j} + 3 \mathbf{k}\}$ m
   B) $\{-3 \mathbf{i} - 3 \mathbf{j} - 3 \mathbf{k}\}$ m
   C) $\{5 \mathbf{i} + 7 \mathbf{j} + 9 \mathbf{k}\}$ m
   D) $\{-3 \mathbf{i} + 3 \mathbf{j} + 3 \mathbf{k}\}$ m
   E) $\{4 \mathbf{i} + 5 \mathbf{j} + 6 \mathbf{k}\}$ m

2. A force vector, $\mathbf{F}$, directed along a line defined by PQ is given by
   A) $(\mathbf{F}/ \mathbf{F}) \mathbf{r}_{PQ}$       B) $\mathbf{r}_{PQ}/\mathbf{r}_{PQ}$
   C) $\mathbf{F}(\mathbf{r}_{PQ}/\mathbf{r}_{PQ})$   D) $\mathbf{F}(\mathbf{r}_{PQ}/\mathbf{r}_{PQ})$
**Today’s Objective:**
Students will be able to use the vector dot product to:

a) determine an angle between two vectors, and

b) determine the projection of a vector along a specified line.

**In-Class Activities:**
- Check Homework
- Reading Quiz
- Applications/Relevance
- Dot product - Definition
- Angle Determination
- Determining the Projection
- Concept Quiz
- Group Problem Solving
- Attention Quiz
1. The dot product of two vectors $P$ and $Q$ is defined as
   A) $P \cdot Q \sin \theta$  
   B) $P \cdot Q \cos \theta$  
   C) $P \cdot Q \tan \theta$  
   D) $P \cdot Q \sec \theta$

2. The dot product of two vectors results in a ________ quantity.
   A) Scalar  
   B) Vector  
   C) Complex  
   D) Zero
APPLICATIONS

If you know the physical locations of the four cable ends, how could you calculate the angle between the cables at the common anchor?
For the force $F$ being applied to the wrench at Point A, what component of it actually helps turn the bolt (i.e., the force component acting perpendicular to arm AB of the pipe)?
DEFINITION

The dot product of vectors $A$ and $B$ is defined as $A \cdot B = A \cdot B \cos \theta$.
The angle $\theta$ is the smallest angle between the two vectors and is always in a range of $0^\circ$ to $180^\circ$.

**Dot Product Characteristics:**
1. The result of the dot product is a scalar (a positive or negative number).
2. The units of the dot product will be the product of the units of the $A$ and $B$ vectors.
Examples: By definition, \( i \cdot j = 0 \)

\[
i \cdot i = 1
\]

\[
A \cdot B = (Ax \ i + Ay \ j + Az \ k) \cdot (Bx \ i + By \ j + Bz \ k)
\]

\[
= Ax \ Bx + Ay \ By + Az \ Bz
\]
USING THE DOT PRODUCT TO DETERMINE THE ANGLE BETWEEN TWO VECTORS

For these two vectors in Cartesian form, one can find the angle by

a) Find the dot product, \( \mathbf{A} \cdot \mathbf{B} = (A_x B_x + A_y B_y + A_z B_z) \),

b) Find the magnitudes (A & B) of the vectors \( \mathbf{A} \) & \( \mathbf{B} \), and

c) Use the definition of dot product and solve for \( \theta \), i.e.,

\[
\theta = \cos^{-1} \left[ \frac{(\mathbf{A} \cdot \mathbf{B})}{(A \cdot B)} \right], \text{ where } 0 \leq \theta \leq 180^\circ.
\]
DETERMINING THE PROJECTION OF A VECTOR

Steps:
1. Find the unit vector, \( \mathbf{u}_{aa} \) along line \( aa' \)
2. Find the scalar projection of \( \mathbf{A} \) along line \( aa' \) by
   \[
   A|| = \mathbf{A} \cdot \mathbf{u}_{aa} = AxUx + AyUy + AzUz
   \]

You can determine the components of a vector parallel and perpendicular to a line using the dot product.
3. If needed, the projection can be written as a vector, $A||$, by using the unit vector $uaa'$ and the magnitude found in step 2.

$$A|| = A|| uaa'$$

4. The scalar and vector forms of the perpendicular component can easily be obtained by

$$A \perp = (A^2 - A||^2)^{\frac{1}{2}}$$

and

$$A \perp = A - A||$$

(rearranging the vector sum of $A = A\perp + A||$)
EXAMPLE 9

**Given:** The force acting on the hook at point A.

**Find:** The angle between the force vector and the line AO, and the magnitude of the projection of the force along the line AO.
\( \mathbf{F} = \{-6 \mathbf{i} + 9 \mathbf{j} + 3 \mathbf{k}\} \text{ kN} \)
CONCEPT QUIZ

1. If a dot product of two non-zero vectors is 0, then the two vectors must be ____________ to each other.
   A) Parallel (pointing in the same direction)
   B) Parallel (pointing in the opposite direction)
   C) Perpendicular
   D) Cannot be determined.

2. If a dot product of two non-zero vectors equals -1, then the vectors must be ____________ to each other.
   A) Collinear but pointing in the opposite direction
   B) Parallel (pointing in the opposite direction)
   C) Perpendicular
   D) Cannot be determined.
GROUP PROBLEM SOLVING 4

Given: The 300 N force acting on the bracket.

Find: The magnitude of the projected component of this force acting along line OA
ATTENTION QUIZ

1. The dot product can be used to find all of the following except ____ .
   A) sum of two vectors
   B) angle between two vectors
   C) component of a vector parallel to another line
   D) component of a vector perpendicular to another line

2. Find the dot product of the two vectors \( P \) and \( Q \).
   \[ P = \{5 \, i + 2 \, j + 3 \, k\} \text{ m} \]
   \[ Q = \{-2 \, i + 5 \, j + 4 \, k\} \text{ m} \]
   A) \(-12 \text{ m}^2\)  
   B) \(12 \text{ m}^2\)  
   C) \(12 \text{ m}^2\)  
   D) \(-12 \text{ m}\)  
   E) \(10 \text{ m}\)